Solutions

6.1: Sets and Set Operations

The theory of sets is the foundation for most of mathematics. In 1922, Ernst Zermelo and Abraham Fraenkel established the first axiomatic approach to sets and these foundations live on today known as Zermelo-Fraenkel Set Theory, or just Set Theory for short. We will obey the axioms that these mathematicians set out, although we will not discuss the axioms specifically, but more in general terms.

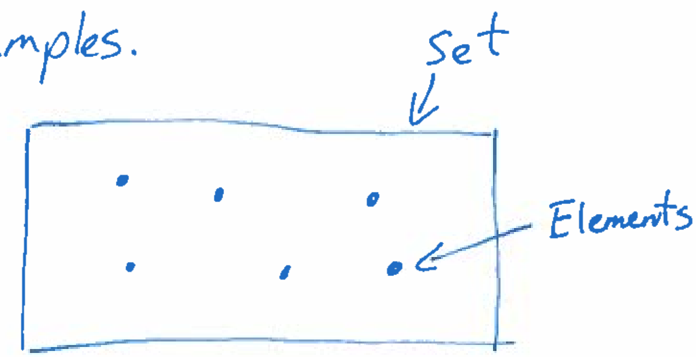
The first question is "what is a set?" How can you think about and visualize them? George Cantor once defined a set as

> "any collection into a whole of definite and separate objects of our intuition or of our thought."

Exercise 1. Take a minute to disect this definition and to visualize a set in your mind. Use the space below to draw a picture of your visualization. Leave space so that you can add others visualization as well.

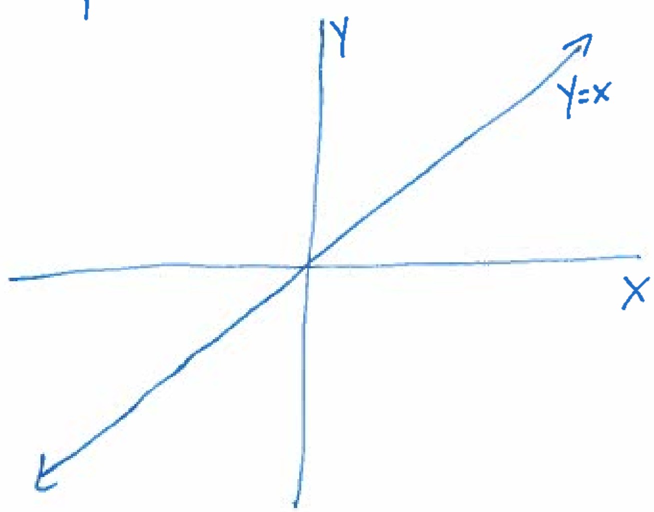
Fill this out yourself. Below are a few examples.

(a)



(b) The plane $\mathbb{R}^2$ is the set consisting of all pairs (x,y) such that $x \in \mathbb{R}$ and $y \in \mathbb{R}$.

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) : x, y \in \mathbb{R}\}$$



(c) The line $y=x$ is the set of solutions to the equation. Let us denote it by A.

$$A = \{(x,y) \in \mathbb{R}^2 : y = x\} = \{(x,x) \in \mathbb{R}^2 : x \in \mathbb{R}\}.$$

Clearly $A \subseteq \mathbb{R}^2$.

Definitions and Notation

1. A Set is a collection of items. The items in the set are referred to as Elements.

2. If A is a set and x is one of the elements in A, we say that x is an element of A and we write $x \in A$.

3. If A and B are two sets which have the same elements, then we say B **is equal to** A and write $B = A$.

4. If A and B are two sets and every elements of B is also an element of A, then we say B is a subset of A and write $B \subseteq A$. Furthermore, if there is at least one element of A which is not contained in B, then we say B is a proper subset of A and write $B \subset A$. Notice that these concepts and their notations are similar to those of inequalities and strict inequalities.

5. The **empty set** is the set containing no elements. It is denoted by $\emptyset$ and is a subset of every set.

6. We say that a set is **finite** if it contains finitely many elements. A set is called **infinite** if it does not contain finitely many elements.

7. When performing an experiment (or activity), we will often refer to the Sample Space, usually denoted by S. This set can be thought of as the **set of outcomes** of the experiment and contains as elements all possible outcomes.

Set-Builder Notation

Example 1. Consider the set $B = \{0, 2, 4, 6, 8\}$. We can describe this set in words:

B is the set of all n such that n is a nonnegative even integer less than 10.

In set-builder notation we write

$$B = \{n : n \text{ is a nonnegative even integer less than } 10\}$$

or

$$B = \{n \in 2\mathbb{Z} : 0 \leq n < 10\}.$$